Modeling Acoustic Wave Behavior in Shale-Sandstone Systems:

2D Discrete Approach with Real-World Variability

Abstract

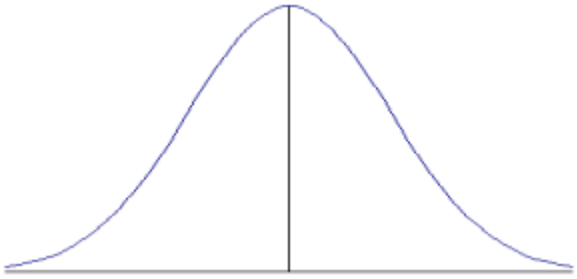
In this project, we model the propagation of P-waves through a two-dimensional subsurface medium made up of shale and sandstone layers. A discrete finite-difference approximation of the 2D wave equation simulates seismic wave behavior across a grid representing 1000 feet of depth and 1000 feet across. The model uses an initial Gaussian pulse to simulate a seismic source. Each rock layer has unique velocity and density properties, and the simulation includes a visual overlay to distinguish shale and sandstone intervals in real time. This project explores how wave amplitudes and travel times are affected by alternating lithologies, emphasizing the relationship between material properties and seismic energy transmission.

Introduction

Understanding seismic wave behavior in layered geological media is essential for geophysics and energy exploration. P-waves, also called primary or compressional waves, travel at different speeds depending on the elastic properties and densities of the materials they pass through. Shale and sandstone are two common sedimentary rocks with contrasting acoustic impedances. In this study, we build a 2D MATLAB simulation that visualizes how seismic energy propagates through the layered media. By simulating the wave propagation over time and comparing the interactions with different lithologies, we show how stratigraphy impacts seismic wave behavior.

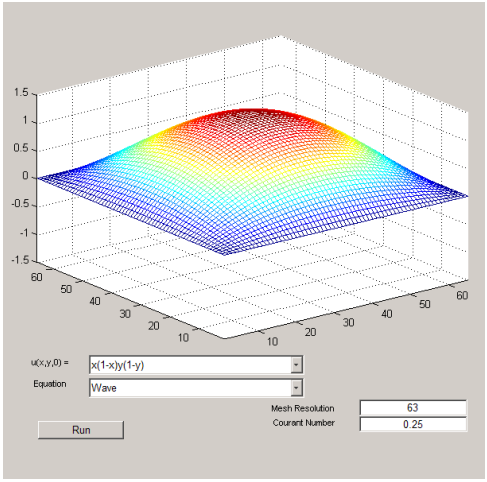
Methods

We create a 2D array of 1000x1000 points, each point representing a location in the subsurface. An initial disturbance is applied as a Gaussian pulse at a given location. We are simulating the initial energy burst that generates compressional P-waves. These waves then travel through the subsurface, reflecting and refracting at boundaries like the shale sandstone interface.



Gaussian Pulse

At each grid point, we numerically solve the 2D wave equation using the finite-difference time-domain (FDTD) method. The second-order central difference approximation is used for both time and spatial derivatives.



2D Wave Equation Model Example

Shale and sandstone layers are implemented by defining depth intervals, each with a different P-wave velocity. Shale is modeled with a lower velocity (2,750 m/s) while sandstone is assigned a higher velocity (4,000 m/s). The simulation runs for 900 time steps. Every 25 steps, a snapshot is rendered using MATLAB's imagesc, and a shale (top) and sandstone (bottom) color overlay is drawn on the figure to indicate the lithology at varying depths.

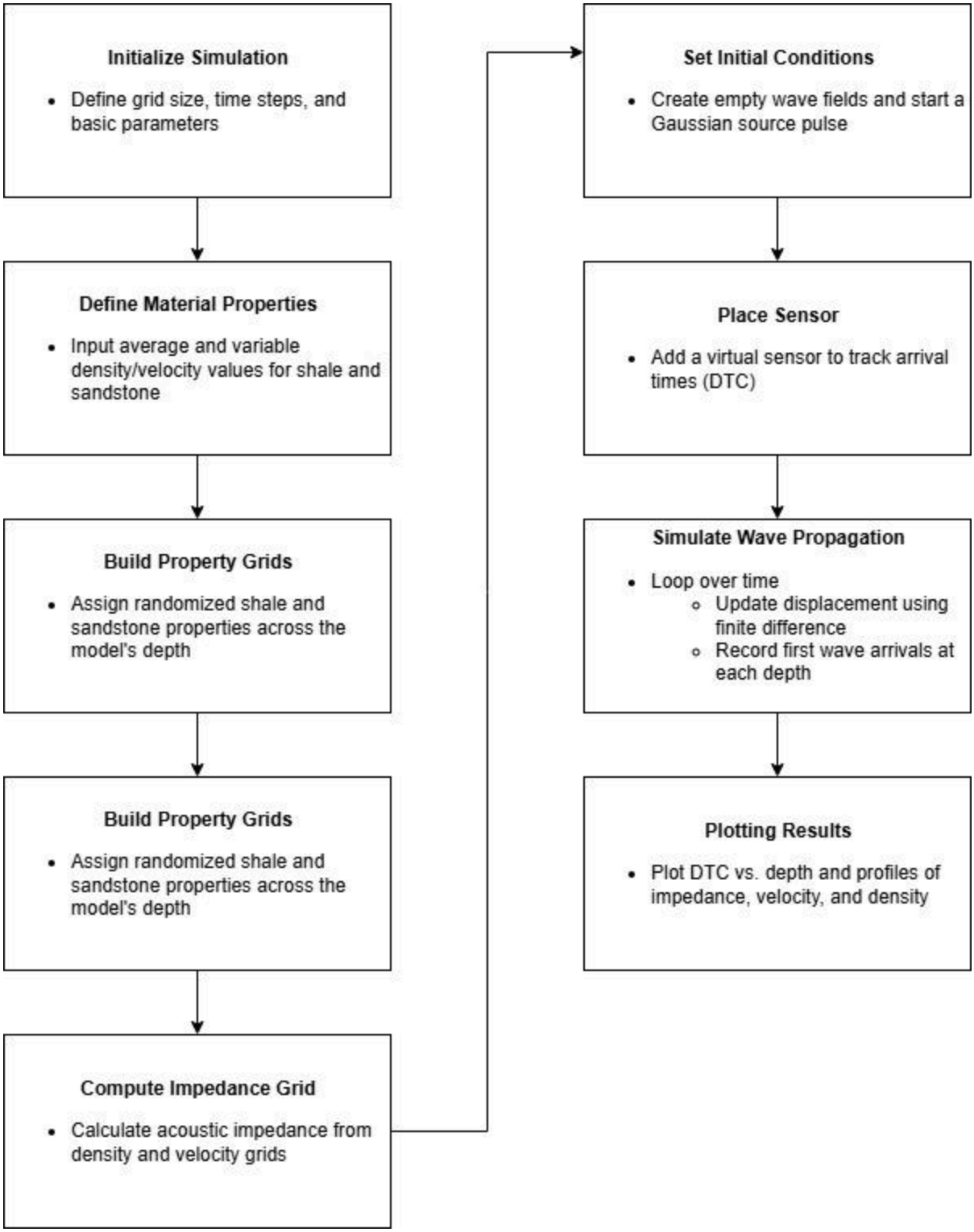
Physical Methods:

Acoustic Wave Propagation	$\frac{\partial^2 u}{\partial t^2} = v^2\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$
Material Impedance	$Z = \rho \cdot v$
Sonic Transit Time	$\mathrm{DTC} = \frac{\Delta z}{v}$

Mathematical Methods:

Finite Difference Approximation of the 2D Wave Equation:	$\frac{\partial^2 u}{\partial x^2} \approx \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{\Delta x^2}$
	$\frac{\partial^2 u}{\partial y^2} \approx \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{\Delta y^2}$
$u_{t+1}(i,j)$	$= 2u_t(i,j) - u_{t-1}(i,j) + v(i,j)^2 \cdot \Delta t^2 \cdot \nabla^2 u(i,j)$

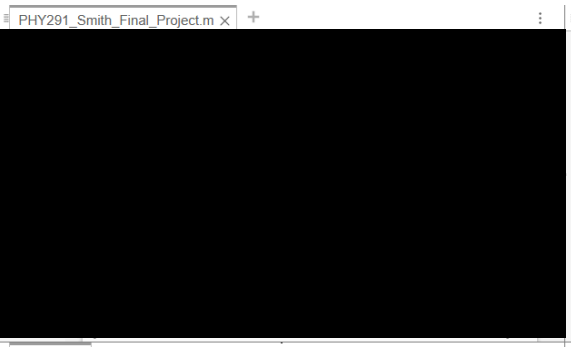
Gaussian Pulse Source Equation:	$u(x,y,0) = A \cdot \exp\left(-\frac{(x-x_0)^2 + (y-y_0)^2}{2\sigma^2}\right)$
Random Property Variability:	$\rho_{i,j} = \rho_{\text{base}} \cdot (1+\epsilon_\rho), \quad v_{i,j} = v_{\text{base}} \cdot (1+\epsilon_v)$
Impedance Averaging for Layer Detection:	$Z_{\text{avg}}(y) = \frac{1}{N_x}\sum_{x=1}^{N_x} Z(x,y)$

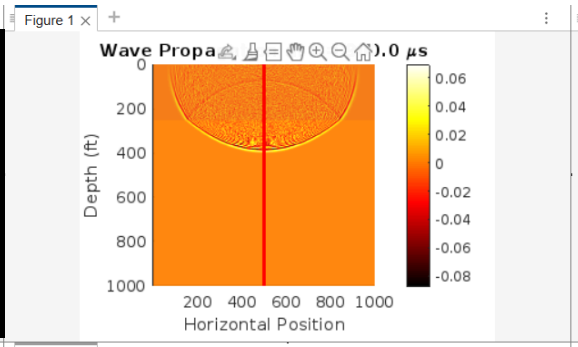

Initialize Simulation
Define grid size, time steps, and basic parameters
Define Material Properties
Input average and variable density/velocity values for shale and sandstone
Build Property Grids
Assign randomized shale and sandstone properties across the model's depth
Build Property Grids
Assign randomized shale and sandstone properties across the model's depth
Compute Impedance Grid
Calculate acoustic impedance from density and velocity grids
Set Initial Conditions
Create empty wave fields and start a Gaussian source pulse
Place Sensor
Add a virtual sensor to track arrival times (DTC)
Simulate Wave Propagation
Loop over time
Update displacement using finite difference
Record first wave arrivals at each depth
Plotting Results
Plot DTC vs. depth and profiles of impedance, velocity, and density

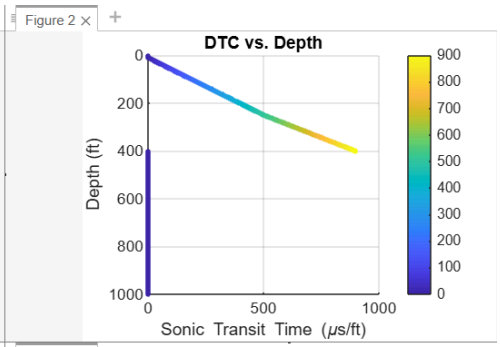
Results

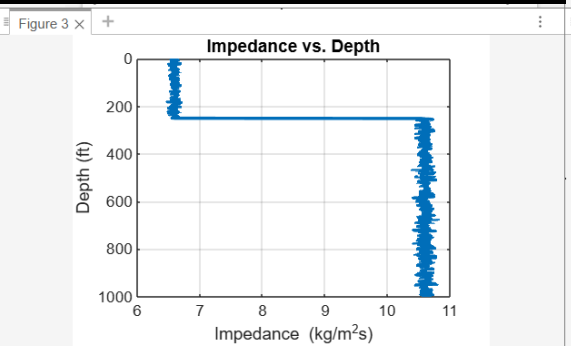
The simulation demonstrates wave propagation through the differing lithologies. The seismic pulse travels downward and outward from the source, and changes in wave velocity at rock boundaries to cause refraction, reflection, and amplitude variations.

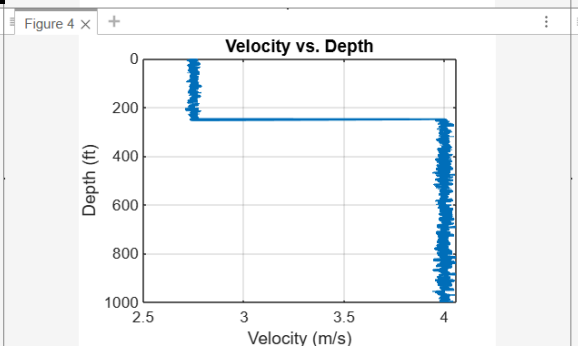
Shale layers have lower velocity and show slower progression compared to sandstone layers. The inclusion of visual overlays allows users to correlate wave behavior with subsurface lithology. The sensor line overlaid on the figure also highlights how wave arrival times differ depending on the path taken through different materials.

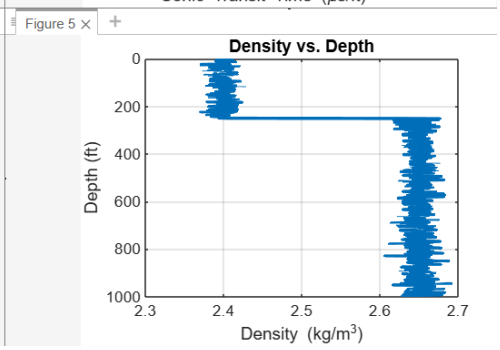








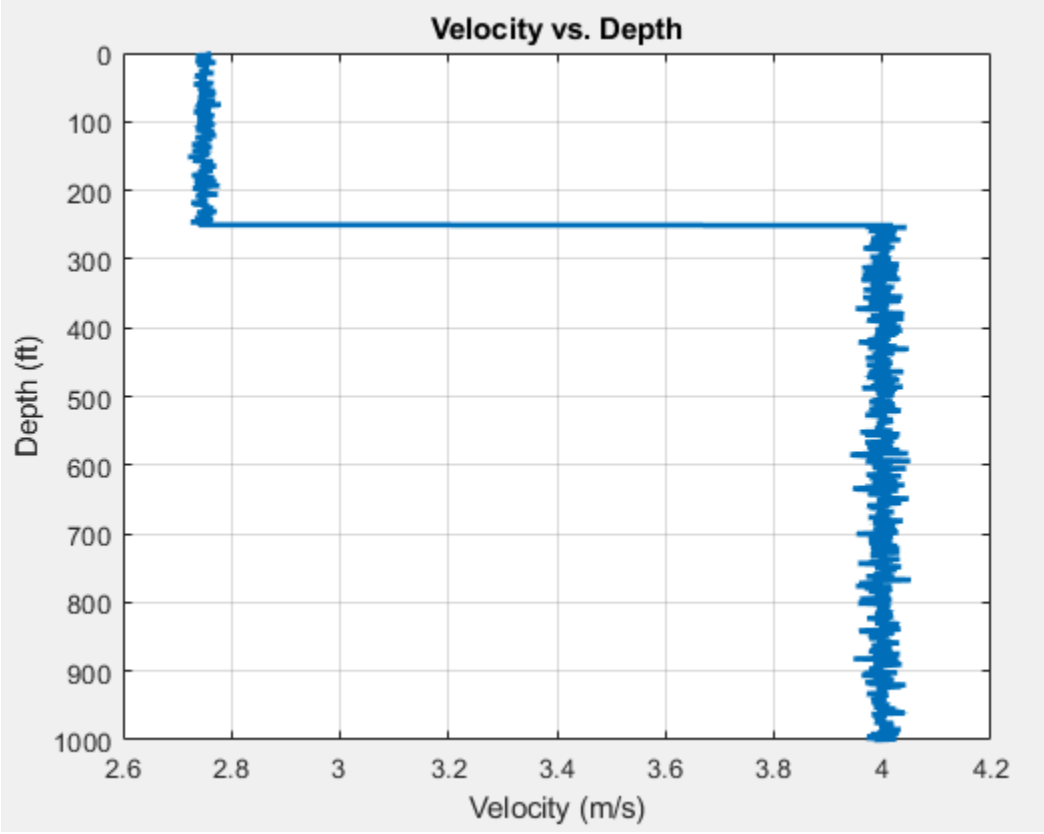




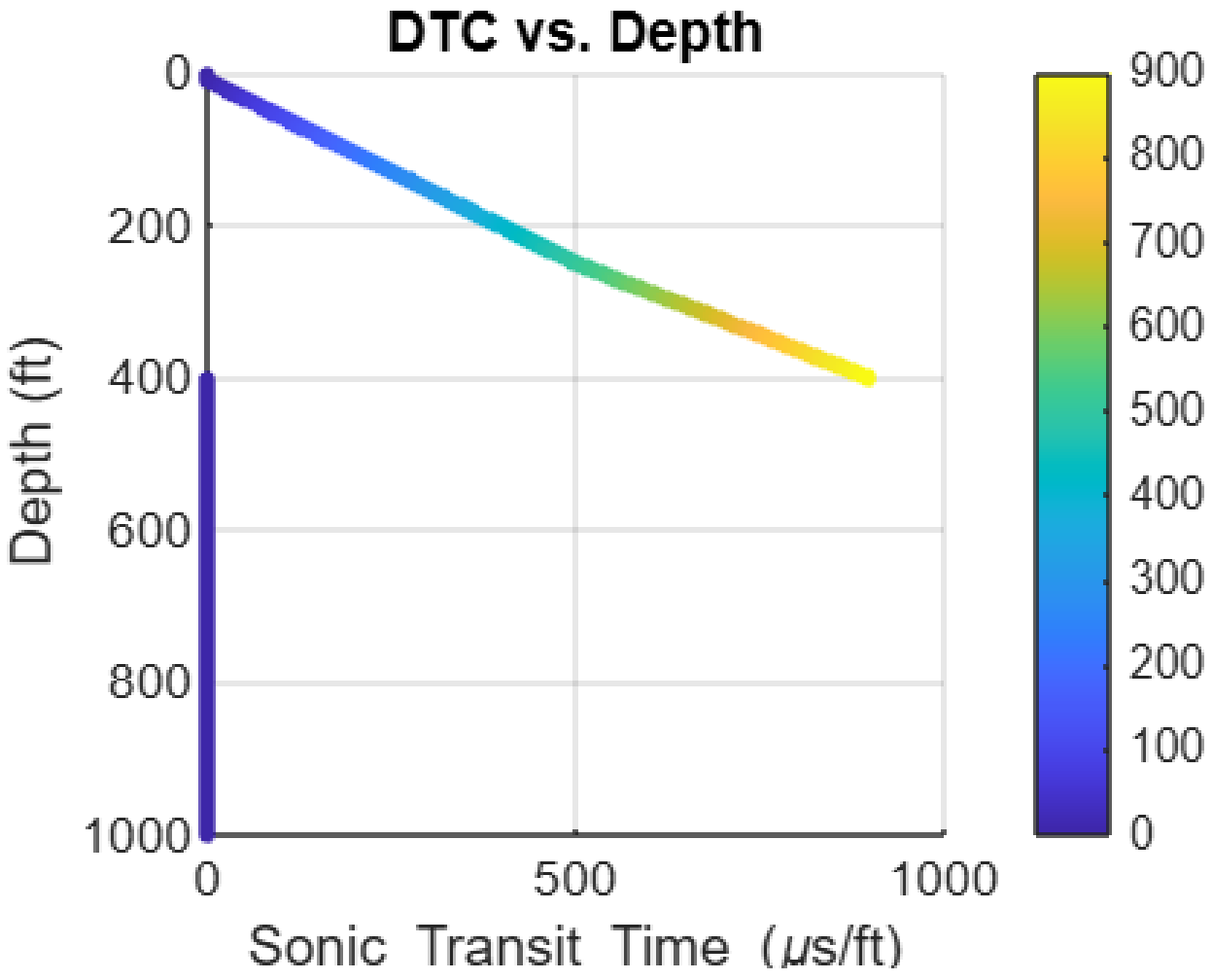
Interpretation

P-wave velocity depends on lithology. In this 2D lattice system, sandstone and shale layers create realistic conditions for simulating seismic wave behavior. Sandstone layers, being denser and more rigid, transmit waves faster, whereas shale layers, being softer and less dense, slow wave propagation and reflect some of the wave energy.

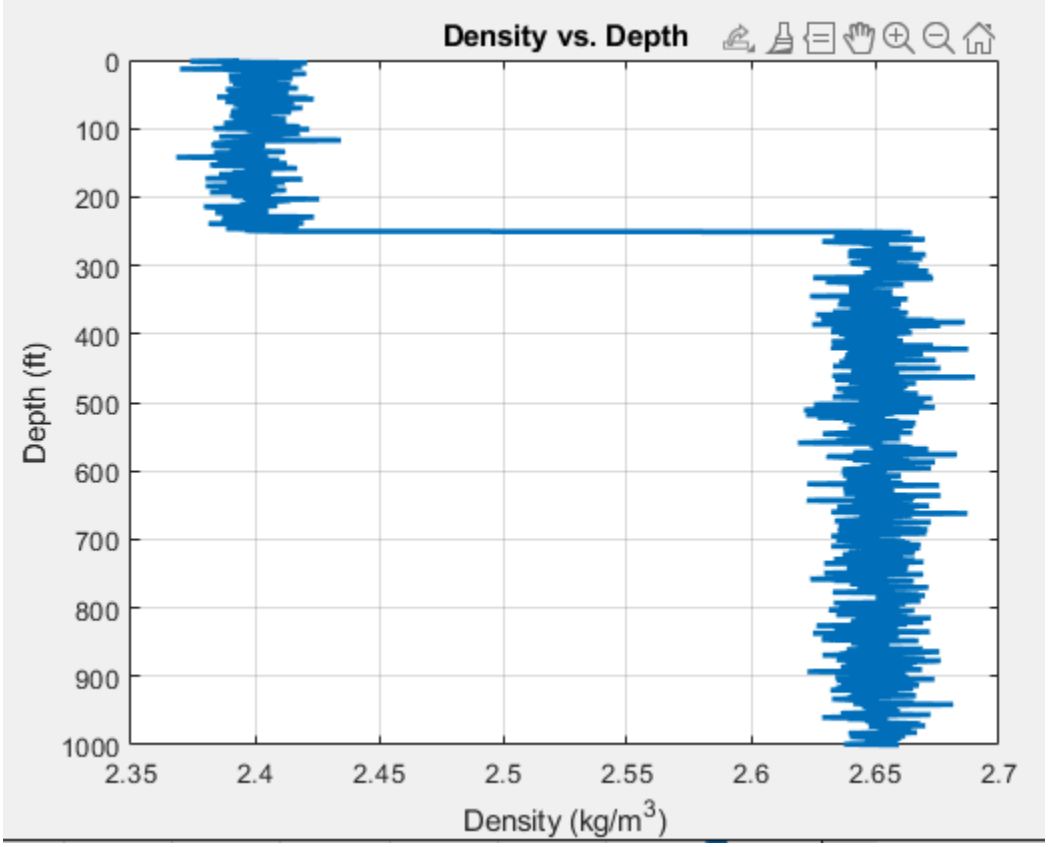
Velocity vs. Depth: The P-wave velocity increases in sandstone layers and decreases in shale layers. This is due to differences in elastic modulus and bulk density. In the model, sandstone was assigned a higher velocity (4,000m/s), while shale had a lower velocity (2,750m/s). This causes waves to accelerate and decelerate as they cross material boundaries.



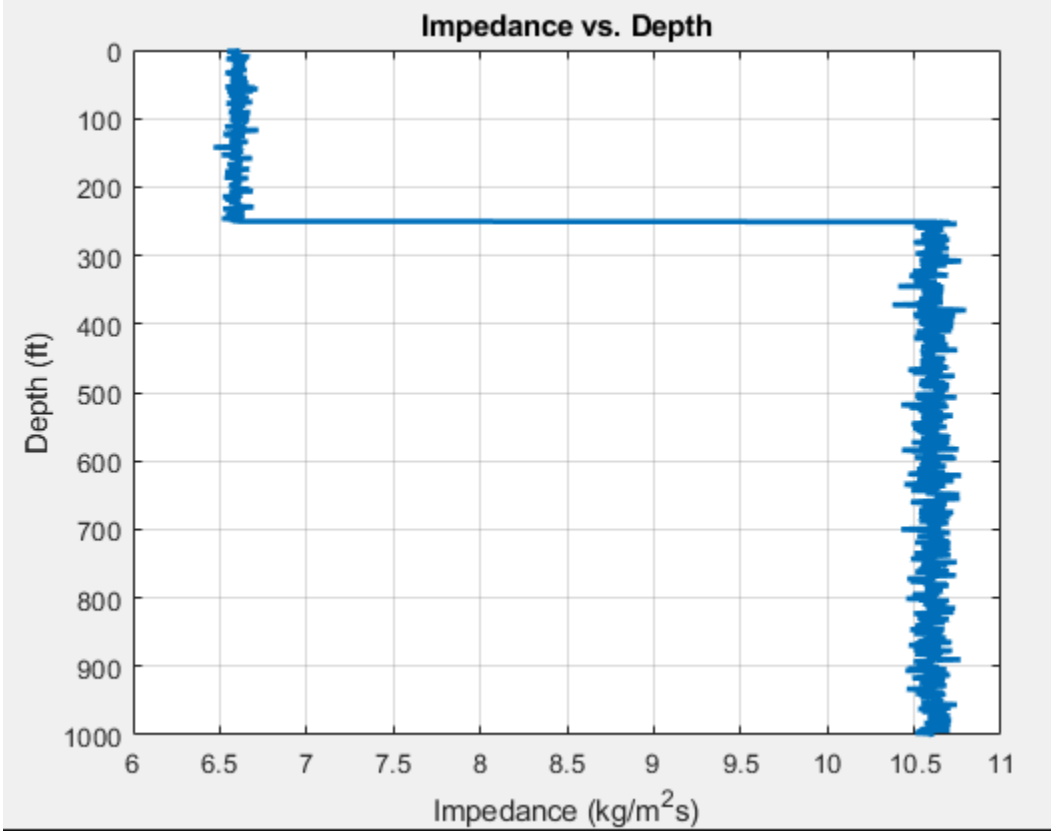
DTC vs. Depth: DTC (compressional sonic transit time, measured in μs/ft) is inversely related to velocity. DTC is lower in fast sandstone layers and higher in slower shale layers. The slope of the line changes from steeper through the shale to a less steep line through the sandstone layer. This difference is more distinct through more widely varied mediums.



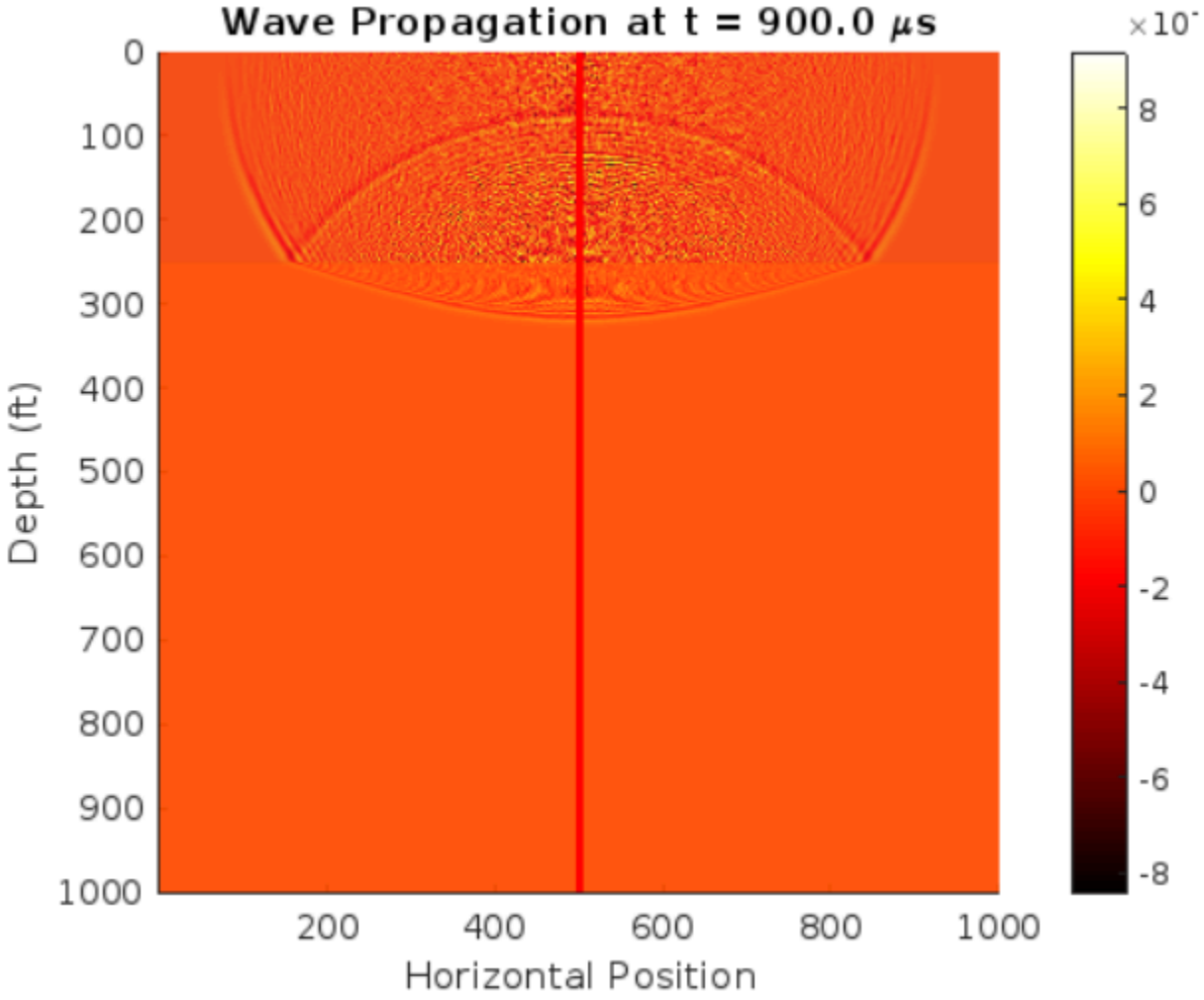
Density vs. Depth: The model assigns different densities to each rock type, higher for sandstone and lower for shale. This models typical geological values where compaction and mineral content make sandstones denser than organic-rich or clay-dominated shales. The data appears more physical thanks to the density randomizer and models the shape of a well-log graph. Though the density contrast is smaller than the velocity contrast, it contributes to the overall behavior of the wave.



Impedance vs. Depth: Seismic impedance is the product of density and velocity ($Z = \rho \times V$). Its variation with depth is what controls reflection strength. In this model, impedance increases in sandstone and decreases in shale. This causes partial reflections of the wavefront at rock boundaries, like the shale-sandstone interface. This is consistent with real seismic reflection principles, where sudden impedance changes produce stronger seismic reflections.



Reflection vs. Boundaries: To better model the reflection of the wave at the boundary point, the sandstone layer densities were changed from 2.65 to 16g/cm^3. With this change, this difference in wave reflection is more than enough to be visible in the model and in the graphic below, this property can be seen at the depth of 250ft, the depth at which the shale transitions to sandstone. At this depth, a portion of the downward-propagating wave is reflected back upward, while the rest continues into the denser material with a change in amplitude and velocity. This models real-world seismic responses and shows how subsurface densities can affect wave responses.



Conclusion

This MATLAB-based simulation of 2D P-wave propagation in a layered shale and sandstone medium offers a useful visualization of seismic energy transmission in complex geology. By observing wave interactions with different rock types, we can better understand seismic responses recorded in real field data. This model may later be expanded further by including anisotropy, absorption, or random heterogeneities to better reflect real subsurface conditions.

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